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MAGNETIZED BIANCHI TYPE-VI_0 COSMOLOGICAL MODEL FOR BAROTROPIC FLUID DISTRIBUTION WITH VARIABLE MAGNETIC PERMEABILITY AND DARK ENERGY

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Abstract: We have investigated Bianchi type-VI_0 cosmological model for barotropic fluid distribution in the presence and absence of magnetic field with dark energy Λ .To obtain the explicit solution of the model, we assume that the magnetic field is in xy plane, therefore the current is flowing along the z-axis. Thus F_{12} is the only non-vanishing component of the electromagnetic field tensor F_{ij}. Also, we suppose expansion θ is proportional to the shear σ .The behaviour of the models from physical and geometrical aspects in presence and absence of magnetic field is also discussed.

Keywords: Bianchi VI_0, magnetized, barotropic fluid, dark energy, perfect fluid, variable magnetic permeability.

1. Introduction

Modern cosmology is concerned with nothing less than a thorough understanding and explanation of past history, the present, and the future evolution of the universe. Friedmann-Robertson-Walker models are the simplest models of the expanding universe which are spatially homogeneous and isotropic where the source of the gravitational field is most naturally considered to be a perfect fluid.

Cosmological research is mainly concerned with time, evolution of various physical parameters. Along with these parameters, in recent year a new physical entity Λ has resurrected in the foreground of cosmology. Cosmologists consider dark energy as the most important factor behind the expansion of the universe. Cosmologists have proposed many candidates for this dark energy, however cosmological constant Λ is the simplest and most hypothetically attractive candidate of it with equation of state $\gamma = -1$.

Bianchi types cosmological models are important in the sense that these are spatially homogeneous and anisotropic from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view established fact today and their importance for the variety of astrophysical phenomena is generally acknowledged as pointed out by Zel'dovich et al. [22]. Also Harrison [7] has suggested that magnetic field could have cosmological origin.

Cosmological model with an incident magnetic field for various Bianchi space times have been investigated by several researchers. Roy and Banerjee [18], Nayak and Bhuyan [11] etc. Tikekar and Patel [19] have obtained some exact solutions of massive strings of Bianchi type-III space time in the presence of magnetic field. Ram and Singh [16] have investigated Bianchi type II, III and IX cosmological model with Matter and electromagnetic fields Bali and Jain [2] have studied Bianchi type III non-static magnetized cosmological model for perfect fluid distribution in General Relativity. Tyagi and Chhajed [20] have investigated Bianchi Type IX cosmological model for perfect fluid distribution with Electromagnetic field. Tyagi et al. [21] investigated Bianchi type-IX string cosmological model with cosmological term Λ in the presence of bulk viscous fluid with electromagnetic field.

A large number of astrophysical observations prove the existence of magnetic fields in galaxies. Galactic magnetic fields which we observe today could be relices of a coherent magnetic field existing in the early universe before galaxy formation. Any theoretical study of cosmological models which contain a magnetic field must take into account that the corresponding universes are necessarily anisotropic.

Here we confine ourselves to models of Bianchi type-VI_0. Bali and Bola[5] have investigated Bianchi VI_0 massive string cosmological model with magnetic field and time dependent vacuum energy density in General Relativity. Bali and Poonia [4] investigated Bianchi Type VI0 Inflationary cosmological model in General Relativity. Ram et al. [14] obtained a spatially homogeneous Bianchi type-VI_0 cosmological model in presence of magnetized anisotropic dark energy within the framework of Lyra geometry. Roy et al. [17] discussed Bianchi VI_0 cosmological model with perfect fluid distribution and magnetic field directed along the axial direction. String cosmology in Bianchi type-VI_0 dusty universe with magnetic field is considered by Amirhashchi [1]. Magnetized Bianchi type-VI_0 barotropic massive string universe for perfect and bulk viscous fluid with decaying vacuum energy density Λ is investigated by Pradhan et al. [12, 13]. Ram [15] investigated Bianchi type-VI_0 space times with perfect fluid source. Bali [3] discussed specially homogeneous Bianchi type VI_0 magnetized bulk viscous massive string cosmological model in General Relativity.

Motivated by the above discussion, we have investigated Bianchi type-VI_0 cosmological model with barotropic fluid distribution and dark energy Λ in the presence and absence of magnetic field.

2. The Metric and Field Equations

We consider homogeneous anisotropic Bianchi Type VI_0 metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2x}B^2dy^2 + e^{2x}C^2dz^2 \tag{1}$$

in which A(t), B(t)and C(t) are cosmic scale functions.

The energy momentum tensor T_i^j in the presence of perfect fluid is defined by

$$T_i^j = (\rho + p)v_i v^j + p g_i^j + E_i^j \tag{2}$$

The electromagnetic field tensor E_i^j given by Lichnerowicz [9] as:

$$E_i^j = \bar{\mu}\left[|h|^2\left(v_i v^j + \frac{1}{2}g_i^j\right) - h_i h^j\right] \quad (3)$$

with $\bar{\mu}$ being the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \in_{ijk\ell} F^{k\ell} v^j \quad (4)$$

where $F^{k\ell}$ is the electromagnetic field tensor and $\in_{ijk\ell}$ the Levi-Civita tensor density. We assume that the current is flowing along the z-axis, so magnetic field is in xy-plane. Thus $h_3 \neq 0, h_1 = h_2 = h_4 = 0$ and F_{12} is the only non-vanishing component of F_{ij}. This leads to $F_{23} = F_{13} = 0$ by virtue of equation (4).

We also find that $F_{14} = 0 = F_{24} = F_{34}$ due to the assumption of infinite electrical conducting of the fluid (Maartens [10]).

A cosmological model which contains a global magnetic field is necessarily anisotropic since the magnetic vector specifies a preferred spatial direction (Bronnikov et al. [6]).

The Maxwell's equation is given by

$$\frac{\partial}{\partial x^j}(F^{ij}\sqrt{-g}) = 0 \quad (5)$$

As, the incident magnetic field is taken along z-axis, therefore with the help of Maxwell's equations (5), the only non-vanishing component of F_{ij} is

$$F_{12} = He^{-2x} \quad (6)$$

Now using (6) in (4) we get

$$h_3 = \frac{HC}{\bar{\mu}AB} \quad (7)$$

$$h^3 = \frac{H}{ABCe^{2x}\bar{\mu}} \quad (8)$$

Finally, we obtain the electromagnetic field tensor, E_i^j as,

$$E_1^1 = E_2^2 = -E_3^3 = -E_4^4 = \frac{H^2}{2A^2B^2\bar{\mu}e^{2x}} \quad (9)$$

here we assumed that magnetic permeability is variable quantity and considered as

$$\bar{\mu} = e^{-2x}$$

Thus $\bar{\mu} \to 0$ as $x \to \infty$ and $\bar{\mu} = 1$ when $x \to 0$

Equation (9) takes the following form

$$E_1^1 = E_2^2 = -E_3^3 = -E_4^4 = \frac{H^2}{2A^2B^2} \tag{10}$$

The Einstein's field equation in the geometrized unit $(c = 8\pi G = 1)$ is given by

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -T_i^j \tag{11}$$

For the line-element (1) lead to the following system of equations:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \frac{1}{A^2} + \Lambda = -\left(p + \frac{H^2}{2A^2B^2}\right) \tag{12}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} - \frac{1}{A^2} + \Lambda = -\left(p + \frac{H^2}{2A^2B^2}\right) \tag{13}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{1}{A^2} + \Lambda = -\left(p - \frac{H^2}{2A^2B^2}\right) \tag{14}$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} - \frac{1}{A^2} + \Lambda = \left(\rho + \frac{H^2}{2A^2B^2}\right) \tag{15}$$

$$\frac{B_4}{B} - \frac{C_4}{C} = 0 \tag{16}$$

where the sub-indices 4 in A, B, C denotes ordinary differentiation with respect to t. The scalar expansion θ and shear σ are given by

$$\theta = v^i_{;i} = \frac{A_4}{A} + \frac{2B_4}{B} \tag{17}$$

$$\sigma = \frac{1}{\sqrt{3}}\left[\frac{A_4}{A} - \frac{B_4}{B}\right] \tag{18}$$

3. Solution of the Field Equations

The field equations (12) to (16) represent a system of five independent equations in six unknowns A, B, C, p, ρ and Λ.we need one additional condition to obtain explicit solutions of the system. We assume that the shear tensor (σ) is proportional to the expansion (θ) which leads to

$$A = B^n \tag{19}$$

Equation (16) leads to

B = CK, where K is an integrating constant (20)

Now, without loss of generality, we suppose K = 1 then equation (20) becomes

B = C

From equation (12) and (14), we get

$$\frac{(n^2-1)B_4{}^2}{B^2} + \frac{(n-1)B_{44}}{B} = \frac{2}{B^{2n}} + \frac{H^2}{B^{2n+2}} \tag{21}$$

$$B_{44} + \frac{(n+1)B_4{}^2}{B} = \frac{2}{(n-1)B^{2n-1}} + \frac{H^2}{(n-1)B^{2n+1}} \tag{22}$$

Let us consider $B_4 = f(B)$ and $B_{44} = ff'$ in equation (22) we get

$$ff' + \frac{(n+1)f^2}{B} = \frac{2}{(n-1)B^{2n-1}} + \frac{H^2}{(n-1)B^{2n+1}} \tag{23}$$

Equation (23) can be written in the form

$$\frac{df^2}{dB} + \frac{2(n+1)f^2}{B} = \frac{4}{(n-1)B^{2n-1}} + \frac{2H^2}{(n-1)B^{2n+1}} \tag{24}$$

On integrating equation (24), we get

$$f^2 = \frac{1}{(n-1)B^{2n-2}} + \frac{H^2}{(n-1)B^{2n}} + \frac{M}{B^{2(n+1)}} \ , (\text{where } n \neq 1) \tag{25}$$

where M is the integrating constant.

From equation (25), we have

$$\int \frac{dB}{\sqrt{\frac{1}{(n-1)B^{2n-2}} + \frac{H^2}{(n-1)B^{2n}} + \frac{M}{B^{2(n+1)}}}} = \int dt + M' = t + M' \tag{26}$$

where M' is the integrating constant. Value of B can be obtained from equation (26)

Hence by appropriate transformation of co-ordinates i.e. B = T, x = X, y = Y and z = Z, metric (1) becomes

$$ds^2 = -\frac{dT^2}{\left[\frac{1}{(n-1)T^{2n-2}} + \frac{H^2}{(n-1)T^{2n}} + \frac{M}{T^{2(n+1)}}\right]} + T^{2n}.dX^2 + e^{-2X}T^2dY^2 + e^{2X}T^2dZ^2 \qquad (27)$$

4. The Geometrical and Physical significance of model in the presence of Electromagnetic Field:

The Pressure and Density for the model (27) are given by

$$p = \frac{(n-2)}{(n-1)T^{2n}} + \frac{H^2(3n-1)}{2(n-1)T^{2n+2}} + \frac{(2n+1)M}{T^{2n+4}} - \Lambda \qquad (28)$$

$$\rho = \frac{(n+2)}{(n-1)T^{2n}} + \frac{3(n+1)H^2}{2(n-1)T^{2n+2}} + \frac{(2n+1)M}{T^{2n+4}} + \Lambda \qquad (29)$$

The scalar of expansion for the flow vector ν^i is given by

$$\theta = (n+2)\left[\frac{1}{(n-1)T^{2n}} + \frac{H^2}{(n-1)T^{2n+2}} + \frac{M}{T^{2(n+2)}}\right]^{1/2} \qquad (30)$$

The scalar of shear for the flow vector ν^i is given by

$$\sigma = \frac{(n-1)}{\sqrt{3}}\left(\frac{1}{(n-1)T^{2n}} + \frac{H^2}{(n-1)T^{2n+2}} + \frac{M}{T^{2(n+2)}}\right)^{1/2} \qquad (31)$$

The dark energy Λ is determined if fluid is known to obey an equation of state of the form

$$p = \gamma\rho\,, \quad \text{where } 0 \le \gamma \le 1 \qquad (32)$$

Equation (32) lead to

$$\Lambda(1+\gamma) = \frac{(n(1-\gamma) - 2(1+\gamma))}{(n-1)T^{2n}} + \frac{H^2(3n(1-\gamma) - (1+3\gamma))}{2(n-1)T^{2n+2}} + \frac{(2n+1)(1-\gamma)M}{T^{2n+4}} \qquad (33)$$

So pressure and energy density are

$$p = \frac{2n\gamma}{(1+\gamma)(n-1)T^{2n}} + \frac{H^2\gamma(1+3n)}{(n-1)(1+\gamma)T^{2n+2}} + \frac{2\gamma(2n+1)M}{(1+\gamma)T^{2n+4}} \qquad (34)$$

$$\rho = \frac{2n}{(n-1)(1+\gamma)T^{2n}} + \frac{H^2(3n+1)}{(n-1)(1+\gamma)T^{2n+2}} + \frac{2(2n+1)M}{(1+\gamma)T^{2n+4}} \tag{35}$$

(where $n \neq 1$)

The dominant energy condition (i) $\rho + p \geq 0$ and the strong energy condition (ii) $\rho + 3p \geq 0$ given by Hawking and Ellis [8] lead to

(i)
$$\frac{2n}{(n-1)T^{2n}} + \frac{H^2(3n+1))}{(n-1)T^{2n+2}} + \frac{2(2n+1)M}{T^{2n+4}} \geq 0 \tag{36}$$

(ii)
$$\left(\frac{2n}{(n-1)(1+\gamma)T^{2n}} + \frac{H^2(3n+1)}{(n-1)(1+\gamma)T^{2n+2}} + \frac{2(2n+1)M}{(1+\gamma)T^{2n+4}}\right)(1+3\gamma) \geq 0 \tag{37}$$

(where $n \neq 1$)

5. The Geometrical and Physical significance of model in the absence of Electromagnetic Field

In the absence of magnetic field the geometry of space time is given by

$$ds^2 = -\frac{dT^2}{\left[\frac{1}{(n-1)T^{2n-2}} + \frac{L}{T^{2(n+1)}}\right]} + T^{2n}.dX^2 + e^{-2X}T^2dY^2 + e^{2X}T^2dZ^2 \tag{38}$$

Where L is the integrating constant.

The pressure and density for the model (38) are given by

$$p = \frac{(n-2)}{(n-1)T^{2n}} + \frac{(2n+1)L}{T^{2n+4}} - \Lambda \tag{39}$$

$$\rho = \frac{(n+2)}{(n-1)T^{2n}} + \frac{(2n+1)L}{T^{2n+4}} + \Lambda \tag{40}$$

The scalar of expansion for the flow vector ν^i is given by

$$\theta = (n+2)\left[\frac{1}{(n-1)T^{2n}} + \frac{L}{T^{2(n+2)}}\right]^{½} \tag{41}$$

The scalar of shear for the flow vector ν^i is given by

$$\sigma = \frac{(n-1)}{\sqrt{3}}\left(\frac{1}{(n-1)T^{2n}} + \frac{L}{T^{2(n+2)}}\right)^{½} \tag{42}$$

The dark energy for the model (38) is given by

$$\Lambda(1+\gamma) = \frac{(n(1-\gamma) - 2(1+\gamma))}{(n-1)T^{2n}} + \frac{(2n+1)(1-\gamma)L}{T^{2n+4}} \tag{43}$$

So pressure and energy density are

$$p = \frac{2n\gamma}{(1+\gamma)(n-1)T^{2n}} + \frac{2\gamma(2n+1)L}{(1+\gamma)T^{2n+4}} \tag{44}$$

$$\rho = \frac{2n}{(n-1)(1+\gamma)T^{2n}} + \frac{2(2n+1)L}{(1+\gamma)T^{2n+4}} \tag{45}$$

(where $n \neq 1$)

The dominant energy condition (i) $\rho + p \geq 0$ and the strong energy condition (ii) $\rho + 3p \geq 0$ given by Hawking and Ellis [8] lead to

(i) $$\frac{2n}{(n-1)T^{2n}} + \frac{2(2n+1)L}{T^{2n+4}} \geq 0 \tag{46}$$

(ii) $$\left(\frac{2n}{(n-1)(1+\gamma)T^{2n}} + \frac{2(2n+1)L}{(1+\gamma)T^{2n+4}}\right)(1+3\gamma) \geq 0 \tag{47}$$

The magnitude of rotation ω is

$$\omega = 0 \tag{48}$$

6. Conclusion

Bianchi type VI_0 cosmological models with dark energy and perfect fluid as the source of matter are obtained in presence and absence of magnetic field. The model presented in this paper could give an appropriate description of the evolution of the universe.

The energy density, pressure, expansion θ and shear σ in the model decrease more rapidly in presence of magnetic field. The model starts expanding with big-bang at $T = 0$. The expansion θ decreases as time increases for $n > 0$ it approaches to zero as $T \to \infty$ and stops when $n = -2$.

In the absence of Magnetic field, the model starts expanding with big bang at $T=0$. The expansion θ is decreasing function of cosmic time T, for $n > 0$ and approaches to zero as $T \to \infty$ and also stops at $n= -2$.

Since $T \to \infty$, $\frac{\sigma}{\theta} = \frac{n-1}{\sqrt{3}(n+2)} \neq 0,$ in both cases, therefore the model does not approach isotropy for large value of T, however it is isotropized for n = 1.

Dark energy Λ for these models are found to be decreasing function of time T for $n > 0$ and approaches to zero at late time, which is in agreement with present astronomical observations. A point type singularity is observed as $T \to 0$, $g_{11} \to 0$, $g_{22} \to 0$ $g_{33} \to 0$ for $n > 0$.Also, we can observe that the energy density and pressure are decreasing function of time and tend to zero as $T \to \infty$, therefore the model essentially gives empty space for large time. The energy conditions given by Hawking and Ellis [8] $\rho + p \geq 0, \rho + 3p \geq 0$ are satisfied.

Hence, in general, the present model represents expanding, shearing and non-rotating universe.

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References

[1] Amirhashchi, H. (2013). String cosmology in Bianchi type-VI$_0$ Dusty universe with electromagnetic field. PRAMANA Journal of Physics, **80**,723-738.

[2] Bali, R. & Jain, S. (2007). Bianchi type–III non static magnetized cosmological model for perfect fluid distribution in General Relativity. Astrophysics Space Science, **31**(1), 401-406.

[3] Bali, R. (2008). Bianchi type-VI$_0$ magnetized bulk viscous massive string cosmological model in General Relativity. Astrophysics and space science, **317**(1), 21-26.

[4] Bali, R. & Poonia, L. (2013). Bianchi type-VI$_0$ inflationary cosmological model in general relativity. International Journal of Modern Physics World Scientific Publishing Company, Conference Series, **22**, 593-602.

[5] Bali, R. & Bola, S.C. (2017). Bianchi-VI$_0$ massive string cosmological model with magnetic field and time dependent vacuum energy density in General relativity. International Journal of Development Research, **12(**7), 17959-17968.

[6] Bronnikov, K.A., Chudayeva, E.N. & Shikin, G.N. (2004).Magneto-dilatonic Bianchi–I cosmology: Isotropization and Singularity Problems. Classical and Quantum Gravity, **21** (14), 3389-3403.

[7] Harrison, E.R. (1973). Origin of Magnetic Fields in the Early Universe. Phy. Rev. Lett., **30**, 188.

[8] Hawking, S.W. and Ellis, G.F.R. (1973). The large scale structure of space-time, Cambridge University Press, Cambridge, **94**.

[9] Lichnerowicz, A. (1967). Relativistic Hydrodynamics and Magneto Hydrodynamics, Benjamin, Elmsford, NY, USA.

[10] Maartens, R. (2000). Cosmological magnetic fields. Pramana, **55** (4), 575-583.

[11] Nayak, B.K. & Bhuyan, G.B. (1987). Bianchi type V perfect fluid models with source free electromagnetic fields. Gen. Relative. Gravit. **19**, 939-948.

[12] Pradhan, A. & Bali, R. (2008). Magnetized Bianchi type-VI_0 Barotropic Massive String Universe with Decaying vacuum energy density Λ. Electronic Journal of Theoretical Physics, **5**, 91-104.

[13] Pradhan, A. & Lata, S. (2011). Magnetized Bianchi type-VI_0 Bulk viscous Barotropic Massive String Universe with Decaying vacuum energy density Λ. Electronic Journal of Theoretical Physics, **8**, 153-168.

[14] Ram, S., Chandel, S. & Verma, M.K. (2017). Spatially homogeneous Bianchi type-VI_0 cosmological model in the presence of magnetized anisotropic dark energy within the framework of Lyra geometry, Prespacetime Journal, **8** (9), 1113-1121.

[15] Ram, S. (1988). Bianchi type-VI_0 space times with perfect fluid source. Journal of Mathematical Physics, **29**, 449.

[16] Ram, S. & Singh, P. (1993). Bianchi type II, III and IX cosmological model with matter and electromagnetic fields. Astrophysics Sci., **201**, 29-33.

[17] Roy, S.R., Singh, J.P. & Narain, S. (1985). Bianchi type-VI_0 cosmological model with perfect fluid distribution and magnetic field directed along the axial direction. Astrophysics and space science, **111** (2), 389-397.

[18] Roy, S.R. & Banerjee, S.K., (1997). A Bianchi type II cosmological model of Petrov type D representingan imperfect fluid with a source free magnetic field, class. Quantum Gravit, **14**, 2845-2851.

[19] Tikekar, R., & Patel, L.K. (1992). Some exact solutions of string cosmology in Bianchi III space. Gen Relativ. Gravit., **24**, 397.

[20] Tyagi, A., & Chhajed, D. (2012). Homogeneous Anisotropic Bianchi Type IX cosmological model for perfect fluid distribution with Electromagnetic Field. American Journal of Mathematics and Statistics, **2** (3), 19-21.

[21] Tyagi, A., Jain, K. & Chhajed, D. (2018). Magnetized bulk viscous Bianchi type-IX string cosmological model with cosmological term Λ. Journal of Environmental Science, Computer Science and Engineering & Technology, **7**, 22-28.

[22] Zel'dovich, Ya. B., Ruzmainkin, A.A. & Sokoloff, D.D. (1983). Magnetic field in Astrophysics, (Gordon and Breach, New York)